NavIncerta Library

The essence of the DeltaLogN method

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1 The method

Suppose we have obtained, in whatever way, the first three moments of a probability distribution that describes some stochastic variable. From the moments we get the mean M (equals the first moment), the standard deviation Σ (equals the square root of the second moment or variance) and the skewness γ (equals the third moment divided by Σ^3).

If there is reason to assume that a lognormal would be a good approximation of the distribution in question, we have the issue that a standard lognormal is defined by two parameters, not by three. This can be addressed by calculating a *shifted* lognormal. The approach is that the standard deviation and the skewness are preserved, but we calculate a shifted mean. We then obtain an offset C between the actual mean and the shifted mean which can be used in later calculations.

Consider the formula for the skew of the lognormal distribution (see section 3.4. in the NavIncerta Library document 'The Lognormal Distribution):

$$\gamma = \frac{\Sigma^3}{M^3} + 3\frac{\Sigma}{M} \tag{1}$$

If M can be solved as a function of Σ and γ, then the shifted mean is obtained. To indicate that we are calculating the shifted mean and not the actual mean M, we denote the variable to be solved for as M'. So we have:

$$\gamma = \frac{\Sigma^3}{M'^3} + 3\frac{\Sigma}{M'} \tag{2}$$

To arrive at a solution we can use Cardano's formula.

If we have a cubic equation in reduced form:

$$x^3 + px + q = 0 \tag{3}$$

then the solution according to the formula of Cardano is:[1]

$$x = \sqrt[3]{\frac{-q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{\frac{-q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} \tag{4}$$

Now we put in the first equation $p = 3$ and $q = -\gamma$ and $x = \frac{\Sigma}{M'}$, then:

$$x = \sqrt[3]{\frac{\gamma}{2} + \sqrt{\frac{\gamma^2}{4} + 1}} + \sqrt[3]{\frac{\gamma}{2} - \sqrt{\frac{\gamma^2}{4} + 1}} \tag{5}$$

$$\frac{\Sigma}{M'} = \sqrt[3]{\frac{\gamma}{2} + \sqrt{\frac{\gamma^2}{4} + 1}} + \sqrt[3]{\frac{\gamma}{2} - \sqrt{\frac{\gamma^2}{4} + 1}} \tag{6}$$

$$M' = \frac{\Sigma}{\sqrt[3]{\frac{\gamma}{2} + \sqrt{\frac{\gamma^2}{4} + 1}} + \sqrt[3]{\frac{\gamma}{2} - \sqrt{\frac{\gamma^2}{4} + 1}}} \tag{7}$$

To summarize:

We have the first three moments of a stochastic variable. We obtain mean M, standard deviation Σ and skewness γ. To fit a lognormal we calculate a shifted mean M'.

[1] See wikipedia, topic Cubic function, look for the section Cardano's method.

$$\boxed{M' = \frac{\Sigma}{\sqrt[3]{\frac{\gamma}{2} + \sqrt{\frac{\gamma^2}{4} + 1}} + \sqrt[3]{\frac{\gamma}{2} - \sqrt{\frac{\gamma^2}{4} + 1}}}}$$

and the shift:

$$\boxed{C = M - M'}$$

2 Applications

A range of applications can be considered. For some of these applications separate papers may be found in the NavIncerta Library.

The tornado method

A tornado diagram for a target metric and its underlying calculations has a base value. For each uncertainty the 'tornado bars' depict the P90, P50 and P10 impacts on the target metric (e.g. NPV), where the P50 impact is set at the base value (i.e zero impact). The base value being a fixed, non-stochastic number, the distribution of the target metric can be obtained by adding up the three first moments of the tornado bar distributions. This will yield the moments of the target metric. The DeltaLogN equations can then be used to arrive at a lognormal as an approximation of the target metric distribution.

Fitting a lognormal to a decision tree

A decision tree is in fact a discrete distribution of a set of values with their probabilities (that add up to 1). From this set of numbers the first three moments can be calculated using the formulas for the discrete distribution. Using the DeltaLogN equations the shifted lognormal can be calculated that fits the tree.

Portfolio analysis

For a portfolio of assets, each asset may have a probability distribution for e.g. value, described by percentile values. For each asset the mean, standard deviation and skew can be calculated, from which the moments are derived. The moments are added up and the DeltaLogN approach can be used to obtain the distribution of the portfolio.

Note that if there are many assets, or the asset distributions are reasonably symmetric, then it may well be fine to use a normal distribution to describe the aggregate of the assets.

Probabilistic budget or aggregate cost estimate

Suppose there are a number of budget or cost items and one would like a distribution of the aggregate. Similar to the portfolio application it is a matter of assigning a probability distribution and calculating the moments for each individual item and aggregating the moments. Again, a lognormal can be fitted using the DeltaLogN approach.